Important Question Mathematics

Ques:-1 Evaluate $\lim_{x \to \sqrt{2}} \frac{\sqrt{x} - \sqrt{2}}{x - 2}$

Sol^n:-1 Rationalise

$$\frac{\sqrt{x} - \sqrt{2}}{x-2} \times \frac{\sqrt{x} + \sqrt{2}}{\sqrt{x} + \sqrt{2}}$$

$$\frac{(\sqrt{x})^2 - (\sqrt{2})^2}{(x-2)(\sqrt{x} + \sqrt{2})}$$

$$\frac{(x-2)^1}{(x-2)_1 (\sqrt{x} + \sqrt{2})} = \frac{1}{\sqrt{x} + \sqrt{2}}$$

Now put $\lim_{x \to \sqrt{2}}$

$$= \frac{1}{\sqrt{2} + \sqrt{2}} = \frac{1}{2\sqrt{2}}$$ Ans

Ques:-2 Evaluate $\lim_{x \to \infty} \frac{(2x+5)(x+3)}{[3x^2 + 2x - 1][x+9]}$

Sol^n:- $\lim_{x \to \infty} \frac{(2x+5)(x+3)}{[3x^2 + 3x + 1x - 1](x+9)}$

$$\frac{(2x+5)(x+3)}{[3x(x+1) - 1(x+1)](x+9)}$$

$$\frac{(2x+5)(x+3)}{(3x-1)(x+1)(x+9)}$$

Now put $\lim_{x \to \infty}$

$$\frac{(2(\infty)+5)(\infty+3)}{(3\infty - 1)(\infty+1)(\infty+9)} = \frac{(0+5)(0+3)}{(0-1)(0+1)(0+9)} = \frac{5 \times 3}{-1 \times 1 \times 9_3} = \frac{5}{-3}$$

$$= -\frac{5}{3}$$

Ques:-3 Differentiate $\sin(2x+1)$

Sol^n:-3 $y = \sin(2x+1)$

Now differentiate

$$\frac{dy}{dx} = \cdot \frac{d}{dx} \sin(2x+1) \quad \frac{d}{dx}(2x+1)$$

$$\frac{dy}{dx} = \cos(2x+1) \cdot (2x1+0)$$

$$= \cos(2x+1) \cdot 2$$

$$= 2\cos(2x+1)$$

$\left[\because \frac{d}{dx} \sin x = \cos x\right]$

Ques:-4 Differentiate $e^x \cdot a^x + 2x^3 - \log x$

Sol^n:-4 $= \frac{d}{dx}\left[e^x a^x + 2x^3 - \log x\right]$

$$= \frac{d}{dx}(ea)^x + \frac{d}{dx} 2x^3 - \frac{d}{dx} \log x$$

$$= (ea)^x \log(ea) + 2 \times 3x^2 - \frac{1}{x}$$

$$= (ea)^x \log(ea) + 6x^2 - \frac{1}{x}$$

Ques:5 Differentiate $\frac{\sin x}{x}$

Sol^n $\frac{d}{dx} = \frac{\sin x}{x}$

$$= \frac{(\sin x)' x - (x)' \sin x}{x^2}$$

$$= \frac{\cos x \cdot x - \sin x}{x^2}$$

$$= \frac{x\cos x - \sin x}{x^2}$$

$\left[\frac{u}{v} = \frac{u'v - v'u}{v^2}\right]$

Ques :- 6 Differentiate $y = (\sin x)^{\cos x}$

Soln :- 6 $\frac{dy}{dx} = (\sin x)^{\cos x}$

Now taking log both side

$\log y = \log (\sin x)^{\cos x}$ $\left[\log m^n = n \log m\right]$

$\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \log(\sin x)$

Now $\left[(U.V)' = U'V + V'U\right]$

$\frac{1}{y}\frac{dy}{dx} = (\cos x)' \log(\sin x) + \log(\sin x)' \cos x$

$= -\sin x \log \sin x + \frac{1}{\sin x}\frac{d}{dx}\sin x \cdot \cos x$

$= -\sin x \log \sin x + \frac{1}{\sin x} \cdot \cos x \cdot \cos x$

$\frac{1}{y}\frac{dy}{dx} = -\sin x \log \sin x + \frac{\cos^2 x}{\sin x}$

$\frac{dy}{dx} = y\left[-\sin x \log \sin x + \frac{\cos^2 x}{\sin x}\right]$

$\frac{dy}{dx} = (\sin x)^{\cos x}\left[-\sin x \log \sin x + \frac{\cos^2 x}{\sin x}\right]$

Ques:-7 Differentiate $\sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x}}}}$ ...

Sol^{n}-7 $\therefore y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x}}}}$ ...

$$y = \sqrt{\sin x + y}$$

Now squaring both side

$$y^2 = \left(\sqrt{\sin x + y}\right)^2$$

$$y^2 = \sin x + y$$

differentiate w.r.t x

$$\frac{d}{dx} y^2 = \frac{d}{dx} (\sin x + y)$$

$$\therefore 2y \frac{dy}{dx} = \cos x + \frac{dy}{dx}$$

$$2y \frac{dy}{dx} - \frac{dy}{dx} = \cos x$$

$$\frac{dy}{dx} [2y - 1] = \cos x$$

$$\frac{dy}{dx} = \frac{\cos x}{2y - 1}$$

Qu 1.8 Differentiate $x^{x^{x\cdots}}$

$$y = x^{x^{x\cdots}}$$

$$y = x^y$$

Now taking log both sides

$$\log y = \log x^y \qquad \left[\begin{array}{l} \log m^n = n \log m \\ (uv)' = u'v + v'u \end{array}\right]$$

Now differentiate

$$\frac{1}{y}\frac{dy}{dx} = y \log x$$

$$\frac{1}{y}\frac{dy}{dx} = (y)' \log x + (\log x)' y$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log x \cdot \frac{1}{x} \cdot y$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{dy}{dx}\log x \cdot \frac{y}{x}$$

$$\left(\frac{1}{y}\frac{dy}{dx} - \frac{dy}{dx}\log x\right) = \frac{y}{x}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - \log x\right) = \frac{y}{x}$$

$$\frac{dy}{dx}\left[\frac{1 - y\log x}{y}\right] = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{y}{x} \times \frac{y}{(1 - y\log x)}$$

$$\frac{dy}{dx} = \frac{y^2}{x - xy\log x}$$

Ques 9 If $y = \log(\sin x) + e^{5x}$ find $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = \frac{d}{dx}\log(\sin x) + \frac{d}{dx}e^{5x}$$

$$= \frac{1}{\sin x}\frac{d}{dx}\sin x + e^{5x}\frac{d}{dx}5x$$

$$= \frac{1}{\sin x}\cdot\cos x + e^{5x}\cdot 5$$

$$= \frac{\cos x}{\sin x} + 5e^{5x}$$

Now again differentiate.

$$\frac{d^2y}{dx^2} = \left\{\frac{(\cos x)'\sin x - (\sin x)'\cos x}{\sin^2 x}\right\} + 5e^{5x}\cdot\frac{d}{dx}5x$$

$$\frac{d^2y}{dx^2} = \left\{\frac{-\sin x\cdot\sin x - \cos x\cdot\cos x}{\sin^2 x}\right\} + 5e^{5x}\cdot 5$$

$$= \left\{-\frac{\sin^2 x - \cos^2 x}{\sin^2 x}\right\} + 25e^{5x}$$

$$= -\left(\frac{\sin^2 x + \cos^2 x}{\sin^2 x}\right) + 25e^{5x}$$

$$= \frac{-1}{\sin^2 x} + 25e^{5x} \qquad \left[\sin^2 x + \cos^2 x = 1\right]$$

$$= -\operatorname{cosec}^2 x + 25e^{5x} \qquad \left[\because \frac{1}{\sin x} = \operatorname{cosec} x\right]$$

Que :-10 The radius of circle increases at the rate of 0.4 cm/sec. What is the increase of circumference?

Sol^{n} :-10 circumference $= 2\pi r$

$= 2\pi \times 0.4$

$= 0.8\pi$ cm/sec.

Que :-11 Find the rate of change per second of volume of ball with respect to its radius $r = 6$cm.

Sol^{n}. 11 Volume of ball $= \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = \frac{4}{3} \times 3\pi r^2$

$= 4\pi r^2$

$= 4 \times \pi \times (6)^2$

$= 4 \times \pi \times 36$

$= 144\pi$ cm^3/sec.

Que :-12 Find all points of maximum and minima value of fun^{n} $f(x) = 6x^3 - 27x^2 + 36x + 6$

Sol^{n} :-12 $\frac{d}{dx} = \frac{d}{dx}\left(6x^3 - 27x^2 + 36x + 6\right)$

$= 18x^2 - 54x + 36 + 0$

$= 18x^2 - 54x + 36$

Again differentiate

$\frac{d^2}{dx} = 36x - 54$

$\frac{dy}{dx} = 0$

$18x^2 - 54x + 36 = 0$

$9[2x^2 - 6x + 4] = 0$

$2x^2 - 6x + 4 = 0$

$2[x^2 - 3x + 2] = 0$

$x^2 - 3x + 2 = 0$

$x^2 - 2x - 1x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$(x-1)(x-2) = 0$

$x = 1, 2$

$\frac{d^2x}{dx} = 0$

$36x^2 - 54 = 0$

$36x^2 = 54$

$x^2 = \frac{54}{36}$ 3 / 2

$x^2 = \frac{3}{2}$

$x = \sqrt{\frac{3}{2}}$

Maximum = 2

Min = $\frac{\sqrt{3}}{2}$

Qus:-10 Find Mean, Median & Mode from following data

X
21
23
25
28
30
32
46
38
48
46
337

$$\bar{X} = \frac{\Sigma X}{N}$$

$$= \frac{337}{10} = 33.7$$

$$\text{Median} = \frac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th}}{2}$$

$$= \frac{5^{th} + 6^{th}}{2}$$

$$= \frac{30 + 32}{2} = \frac{62}{2} = 31$$

Mode = Highest frequency
= 46 (2 times)

Qus:-11 Find Mean, Median & Mode from following observation?

X	V	XY	cf
1	8	8	8
2	10	20	18
3	11	33	29
4	16	64	45
5	20	100	65
6	25	150	90
7	15	105	105
8	9	72	114
9	6	54	120
	120	606	

$$\bar{X} = \frac{\Sigma XY}{\Sigma Y}$$

$$= \frac{606}{120} = 5.05$$

$$\text{Median} = \frac{N}{2} = \frac{120}{2} \quad 60$$

Hence Median = 5

Mode = 6 [Highest frequency]

Qun-15

C.I	f	x	d=x(x-9)	fd	cf
5-10	5	7.5	-15	-75	5
10-15	6	12.5	-10	-60	11
15-20	15	17.5	-5	-75	26
20-25	10	22.5	0	0	36
25-30	5	27.5	5	25	41
30-35	4	32.5	10	40	45
35-40	2	37.5	15	30	47
40-45	2	42.5	20	40	49
	49			-75	

$$\bar{x} = a + \frac{\sum fd}{\sum f}$$

$$= 22.5 + \frac{-75}{49}$$

$$= 22.5 - 1.5$$

$$= 21$$

Median $\frac{N}{2} = \frac{49}{2} = 24.5$

$$= l_1 + \frac{\frac{N}{2} - cf}{f} \times i$$

$$= 15 + \frac{24.5 - 11}{15} \times 5$$

$$= 15 + \frac{13.5}{30}$$

$$= 15 + 4.5 = 19.5$$

Mode $= l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$

$$= 15 + \frac{15-6}{2\times15 - 10 - 6} \times 5$$

$$= 15 + \frac{9}{30-16} \times 5$$

$$= 15 + \frac{9}{14} \times 5 \quad = \quad 15 + \frac{45}{14} = 18$$

Qun: 16 Calculate Rank correlation

X	Y	R_1	R_2	$d(R_1 R_2)$	d^2
45	35	8	9	-1	1
70	90	4	2	2	4
65	70	5	4	1	1
30	40	10	8	2	4
90	95	1	1	0	0
40	45	9	7	2	4
50	60	7	5	2	4
75	80	3	3	0	0
85	30	2	10	-8	64
60	50	6	6	0	0
					82

$$r = \frac{1 - 6\Sigma d^2}{n(n^2-1)}$$

$$= \frac{1 - 6 \times 82}{10(10^2-1)}$$

$$= 1 - \frac{492}{990}$$

$$= 1 - 0.49$$

$$= 0.51$$